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Probabilistic Teleportation of Two-particle Entangled State via a Cluster State

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Abstract: A scheme for probabilistic teleportation of an unknown two-particle entangled state via a four-qubit non-maximally entangled cluster state as quantum channel is proposed. In this scheme, the unknown entangled states can be teleported with a certain probability, which is determined by the smallest two of the coefficients' absolute values of the cluster state. What's more, if quantum channel is maximally entangled cluster state, the probabilistic teleportation becomes usual teleportation.

Key words: quantum teleportation; cluster state; Bell state measurement; unitary transformation
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1 Introduction

Quantum teleportation has been used as an essential resource in quantum information theory. In 1993, Bennett *et al.*[1] suggested a quantum method of teleportation, through which an unknown quantum pure state is teleported from the sender "Alice" to the receiver "Bob", and "Alice" and "Bob" are spatially separated. In the first step of the teleportation process, two spin-1/2 particles are prepared in a maximally entangled state. Then, a joint measurement is performed on the particle to be teleported and one of the correlated pair. Finally, the information about the joint measurement is transmitted to the other observer through a classic channel, and thus he can reconstruct the initial state of the teleported particle on the second particle of the correlated pair. Since then, quantum teleportation has got a tremendous achievement both theoretically and experimentally[2~5], and has important applications in quantum communication[2], quantum calculation[6], quantum cryptography[7], and so on.

The quantum entangled state, which lies at the heart of quantum information theory, is considered to be a fundamental resource of quantum teleportation. The entangled state of three qubits can be classified into GHZ and W states[8]. A lot of applications using these two kinds of state have been proposed, for example, Shi *et al.*[9] have proposed a scheme for teleporting a two-particle entangled state with a GHZ state, CAO *et al.*[10] have proposed a scheme for teleporting two-particle entangled state with a W state, *etc*.

The entangled state of four qubits is more complicated than that in three qubits, a cluster state of four qubits is common and important. Li *et al.*[11] proposed a scheme to teleport two-particle via cluster state, but they only considered maximally entangled cluster state as quantum channel. Dong *et al.*[12] showed that cluster states have some particular characters in the case of $N>3$, for example, the cluster states have the properties both of the GHZ and W states, and they are harder to be destroyed by local operations than GHZ states.

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In this letter, probabilistic teleportation of an unknown two-particle entangled state via a four-qubit non-maximally entangled cluster state as quantum channel was investigated. In the realm of experiment realization, preparation for maximally entangled state is very difficult because entangled state apparatus is not perfect, which is absolutely impossible to produce ideal maximally entangled state [13]. This usually takes to purifying methods to get the maximally entangled state, but that have to need more entanglement resources. The considerable advantage of our proposition is that teleporting an unknown two-particle entangled state only needs a non-maximally entangled cluster state as quantum channel, which greatly reduces the amount of entanglement resources and thus needs less classical bits. Through analysis, we concluded that if the quantum channel were maximally entangled cluster state as Ref. [11], the unknown entangled state could be successfully teleported from the sender (Alice) to the receiver (Bob), namely, the successful probability were 100%; if the quantum channel were the non-maximally entangled cluster state, in order to achieve teleportation of the unknown entangled state, the Bob needs to introduce an auxiliary qubit and perform two unitary transformations in the process, so the probability of successful teleportation is determined by the smallest two of the coefficients' absolute values of the cluster state. So, the scheme in Ref. [11] is only a special case of our proposition.

2 Probabilistic Teleportation of Two-particle Entangled State

Let's describe teleportation of an unknown two-particle entangled state in present section. Supposing Alice has an entanglement particle state, which consists of particles 1 and 2. She wants to teleport the unknown state $|\Psi\rangle_{12}$ to the receiver Bob. The state $|\Psi\rangle_{12}$ may be expressed as:

$$|\Psi\rangle_{12} = (x|00\rangle + y|11\rangle)_{12} \tag{1}$$

where x and y are unknown coefficients, and $|x|^2 + |y|^2 = 1$. Now, a non-maximally entangled cluster state is used as quantum channel between the sender Alice and the receiver Bob, which is

$$|\Psi|_{3456} = (a|0000\rangle + b|0011\rangle + c|1100\rangle - d|1111\rangle)_{3456} \tag{2}$$

where a, b, c, d are unknown coefficients, and $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$. Since a, b, c, d are unknown probabilistic amplitudes of the state, the simplest situation is $|a| = |b| = |c| = |d| = 0.5$, corresponding to the maximally entangled cluster state as quantum channel in Ref. [11]. Generally speaking, the coefficients a, b, c and d are different. Without loss generality, we can assume $|a| > |b| > |c| > |d|$. We suppose particles 1, 2, 3 and 6 belong to Alice, the rest belong to Bob. Thus, the total state of the system can be expressed as:

$$|\Psi\rangle = (x|00\rangle + y|11\rangle_{12}) \otimes (a|0000\rangle + b|0011\rangle + c|1100\rangle - d|1111\rangle)_{3456} \tag{3}$$

To realize the teleportation, Alice operates a Bell state measurement on particles 1 and 3 at first, then the system state collapses into one of following states:

$$_{13}\langle\Phi^{\pm}|\Psi\rangle = \frac{1}{\sqrt{2}}(xa|00000\rangle + xb|00011\rangle \pm yc|11100\rangle \mp yd|11111\rangle)_{2456} \tag{4}$$

$$_{13}\langle\Psi^{\pm}|\Psi\rangle = \frac{1}{\sqrt{2}}(xc|00100\rangle - xd|00111\rangle \pm ya|11000\rangle \pm yb|11011\rangle)_{2456} \tag{5}$$

where $|\Psi^{*}\rangle_{ij}$ and $|\Phi^{\pm}\rangle_{ij}$ are Bell states of the particles pairs (1, 3) and (2, 6),

$$|\Phi^{\pm}\rangle_{ij} = \frac{1}{\sqrt{2}}(|00\rangle_{ij} \pm |11\rangle_{ij}) \tag{6}$$

$$|\Psi^{\pm}\rangle_{ij} = \frac{1}{\sqrt{2}}(|01\rangle_{ij} \pm |10\rangle_{ij}) \tag{7}$$

Secondly, Alice operates a Bell state measurement on particles 2 and 6, the system state collapses into one of following states:

$$_{26}\langle\Phi^{\pm}|_{13}\langle\Phi^{\pm}|\Psi\rangle = \frac{1}{2}(xa|00\rangle \pm^{2} \mp^{1} yd|11\rangle)_{45} \tag{8}$$

$$_{26}\langle\Psi^{\pm}|_{13}\langle\Phi^{\pm}|\Psi\rangle = \frac{1}{2}(xb|01\rangle \pm^{2} \pm^{1} yc|10\rangle)_{45} \tag{9}$$

$$_{26}\langle\Phi^{\pm}|_{13}\langle\Psi^{\pm}|\Psi\rangle = \frac{1}{2}(xc|10\rangle \pm^{2} \pm^{1} yb|01\rangle)_{45} \tag{10}$$

$$_{26}\langle \Psi^{\pm} |_{13}\langle \Psi^{\pm} | \Psi\rangle = \frac{1}{2}(-xd|10\rangle \pm^2 \pm^1 ya|00\rangle)_{45} \quad (11)$$

where, $\pm^1(\mp^1)$ and $\pm^2(\mp^2)$ denote the results corresponding to Bell measurements on particle pairs (1, 3) and (2, 6), respectively.

In order to reconstruct the original state, Bob has to operate corresponding unitary transformations on particles (4,5) according to Alice's measurement results. For example, we suppose Alice's results are $|\Phi^+\rangle_{13}|\Phi^+\rangle_{26}$, Alice informs Bob of her measurement results by means of a classic channel. From that, Bob can conclude that the state of particles 4 and 5 collapse into $\frac{1}{2}(xa|00\rangle - yd|11\rangle)_{45}$. Then, Bob performs a unitary transformation $U_1 = I_4 \otimes Z_5$ on $\frac{1}{2}(xa|00\rangle - yd|11\rangle)_{45}$, where I and Z are the identity operator and the Pauli operator σ_z, respectively. Then, Bob will obtain:

$$_{26}\langle \Phi^+ |_{14}\langle \Phi^+ | \Psi\rangle \rightarrow \frac{1}{2}(xa|00\rangle + yd|11\rangle)_{45} \quad (12)$$

Finally, Bob needs to introduce an auxiliary qubit with the original state $|0\rangle_A$ and operates a unitary transformation under the basis $\{|00\rangle_{4A}, |10\rangle_{4A}, |01\rangle_{4A}, |11\rangle_{4A}\}$:

$$U_2 = \begin{bmatrix} \frac{d}{a} & 0 & 0 & \sqrt{1-\left(\frac{d}{a}\right)^2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sqrt{1-\left(\frac{d}{a}\right)^2} & 0 & 0 & -\frac{d}{a} \end{bmatrix}$$

Under this transformation, Eq. (12) becomes:

$$\frac{1}{2}d(x|00\rangle + y|11\rangle)_{45}|0\rangle_A + \frac{1}{2}\sqrt{1-\left(\frac{d}{a}\right)^2}xa|00\rangle_{45}|1\rangle_A \quad (13)$$

Then, Bob measures the state of particle A. If the result is $|1\rangle_A$, the teleportation fails; if the result is $|0\rangle_A$, the state of particles 3, 5 and 7 will collapse into $\frac{1}{2}d(x|00\rangle + y|11\rangle)_{45}$, the teleportation successfully realized with the probability being $\frac{1}{4}|d|^2$. For Eq. (8), the total probabilities of successful teleportation are $4\times\frac{1}{4}|d|^2 = |d|^2$.

For other Alice's measurement results, in a similar way, Bob should perform the corresponding two unitary transformations U_1 and U_2 on particles (4, 5) , U_1 are shown in Table 1, for the sake of saving space we don't write out all the U_2 (but it is easy to write out). It is easy to see, synthesizing all cases (16 kinds in all), the total probabilities of successful teleportation being $2|c|^2 + 2|d|^2$. If we assume $|d| > |c| > |b| > |a|$, through analysis, the total probabilities of successful teleportation are $2|a|^2 + 2|b|^2$. It is obvious that the smallest two of the coefficients' absolute values of the cluster state determine the total probabilities of successful teleportation. If $|a| = |b| = |c| = |d|$, so $|c| = |d| = 0.5$, corresponding to the maximally entangled cluster state as quantum channel in Ref. [11], thus the total probability is equals to 1.0, this is usual teleportation. It is to say the scheme in Ref. [11] only is a special case of our proposition.

Table 1 The unitary transformations corresponding to Alice's measurement results

Alice's results	Bob's operations	Alice's results	Bob's operations
$\|\Phi^+\rangle_{13}\|\Phi^+\rangle_{26}$, $\|\Phi^-\rangle_{13}\|\Phi^-\rangle_{26}$	$I_4\otimes Z_5$	$\|\Psi^+\rangle_{13}\|\Phi^+\rangle_{26}$, $\|\Psi^-\rangle_{13}\|\Phi^-\rangle_{26}$	$X_4\otimes I_5$
$\|\Phi^+\rangle_{13}\|\Phi^-\rangle_{26}$, $\|\Phi^-\rangle_{13}\|\Phi^-\rangle_{26}$	$I_4\otimes I_5$	$\|\Psi^+\rangle_{13}\|\Phi^-\rangle_{26}$, $\|\Psi^-\rangle_{13}\|\Phi^+\rangle_{26}$	$Y_4\otimes I_5$
$\|\Phi^+\rangle_{13}\|\Psi^+\rangle_{26}$, $\|\Phi^-\rangle_{13}\|\Psi^-\rangle_{26}$	$I_4\otimes X_5$	$\|\Psi^+\rangle_{13}\|\Psi^+\rangle_{26}$, $\|\Psi^-\rangle_{13}\|\Psi^-\rangle_{26}$	$X_4\otimes Y_5$
$\|\Phi^+\rangle_{13}\|\Psi^-\rangle_{26}$, $\|\Phi^-\rangle_{13}\|\Psi^+\rangle_{26}$	$I_4\otimes Y_5$	$\|\Psi^+\rangle_{13}\|\Psi^-\rangle_{26}$, $\|\Psi^-\rangle_{13}\|\Psi^+\rangle_{26}$	$X_4\otimes X_5$

3 Conclusion

In conclusion, we proposed a scheme of probabilistic teleportation of an unknown two-particle entangled state via a four-particle non-maximally entangled cluster state as quantum channel. In this proposal, through Alice's two operations of Bell state measurements and Bob's two operations of unitary transformations in the process, the entangled states can be teleported with a certain probability, which is determined by the smallest two of the coefficients' absolute values of the cluster state. The considerable advantage of our proposition is that teleporting an unknown two-particle entangled state only needs a non-maximally entangled cluster state as quantum channel, which greatly reduces the amount of entanglement resources and thus needs less classical bits. If quantum channel were a maximally entangled cluster state, the probabilistic teleportation becomes usual teleportation, namely, the probability of successful teleportation is equal to 100% as Ref. [11], which is only a special case of our protocol. This is to say that our proposition is more general as compareing with Ref. [11].

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通过 cluster 态实现两粒子纠缠态的量子几率隐形传态

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摘要: 提出通过一个四粒子 cluster 非最大纠缠态作为量子信道来实现一未知两粒子纠缠态的量子几率隐形传态方案。在此方案中,纠缠态可以实现一定的几率传输,此几率由 cluster 态中绝对值较小的两个系数决

定。如果我们用cluster最大纠缠态作为量子信道,此时几率隐形传态就成了一般的隐形传态,即成功传输的几率为100%。

关 键 词: 量子几率隐形传态; cluster态; Bell态测量; 幺正变换

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