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Absorption Property in Rectangular Waveguide with Left-handed Material

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Abstract: In this paper, the absorption property of the left-handed material (LHM) rectangular waveguide was studied by use of the perturbation method, and other novel properties were revealed. Due to the restriction of the dispersion equations, the LHM waveguide can not support the fundamental modes, and its absorption property is quite different from that of RHM (right-handed material) rectangular waveguide. It was shown that E_{11}^{y} mode's attenuation coefficient changes from large to small as the length to width ratio (a/b) grows, while the result is reverse in RHM rectangular waveguide. In addition, when different modes are propagating in the same LHM waveguide there is an absorption peak for every mode, and in a small neighborhood of this peak, an attenuation trough can be found. In this case, higher-order modes possess larger attenuation coefficient than lower-order modes. But, when different modes are propagating in the RHM waveguide the attenuation curves are changing vibrantly and approach the core attenuation coefficient with the increasing of the width b. Otherwise, higher-order modes possess smaller attenuation coefficients than lower-order modes in RHM waveguide.

Key words: absorption peak; attenuation trough; attenuation coefficient; perturbation method; rectangular waveguide
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1 Introduction

Left-handed materials (LHM) have opened up a unique possibility to investigate novel physical phenomena including refraction of waves. Such material, which possesses simultaneously negative values of the dielectric permittivity and magnetic permeability μ, was theoretically analyzed by Veselago in 1968[1]. Despite the physical significance of his analysis, the results appeared to be of limited practical application due to the absence of naturally occurring LHM. Until 2001, the phenomenon of the negative refraction was firstly experimentally verified by using artificial left-handed materials realized by periodic arrays of split ring resonators and wire strips in the microwave region[2].

Then, a great deal of attention[3,4] has been paid to this promising area of research. For example, LHM waveguides, including planar waveguide[5,6], rectangular waveguide[7] and fiber waveguide[8], have been studied. LHM rectangular waveguides are the important devices of integrated optics, they are also the basic structures of semiconductor lasers, modulators, direction couplers, *et al*, and therefore they deserve more attention and further studies. But at present, most of the studies focus on the dispersion curves, the distribution of fields and the propagation properties of light, the absorption property of LHM rectangular waveguide is seldom considered. In fact, most of the LHMs have complex refractive indices with unnegligible imaginary parts, which cause the loss of light. In this paper, we investigated the

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absorption property of LHM rectangular waveguides with complex refractive index, and a numerical analysis was given about the attenuation curves of rectangular waveguide with a LHM core. The expressions of the effective refractive index and the attenuation coefficient were derived by the perturbation method, and a detailed analysis was presented for the example of defined LHM rectangular waveguides in two situations. For the first one, the length to width ratio (a/b) of the waveguide is different, and for the second one, the modes propagating in the waveguide is different.

2 Theoretical Analysis of the LHM Rectangular Waveguide

We consider a rectangular waveguide with the geometry and parameters given in Fig. 1, where a is the length and b is the width of the waveguide. The dielectric permittivity and the magnetic permeability in the core ($i=1$) and the cladding ($i=2,3,4,5$) are related to their vacuum values $\bar{\varepsilon}_i=\varepsilon_0\varepsilon_i-jb_i$ and $\bar{\mu}_i=\mu_0\mu_i-jd_i$ ($i=1,2,3,4,5$), respectively. It is assumed that the cladding has a positive index so that both ε_i and μ_i ($i=2,3,4,5$) are positive, and set the core to have ε_1 and μ_1 negative. Then we assume $\bar{n}_i$ is the complex refractive index of every section, and n_i, κ_i and α_i ($\alpha_i=k_0\kappa_i$) ($i=1,2,3,4,5$) are its real refractive index, extinction coefficient and attenuation coefficient ($\bar{n}_i=n_i-j\kappa_i$).

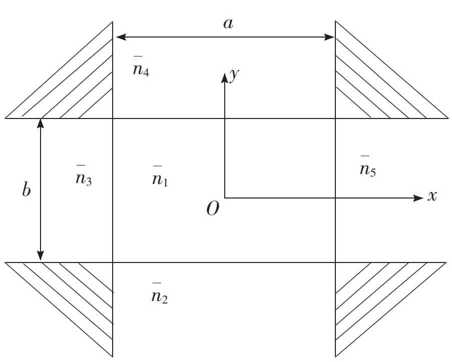

Fig. 1 The cross section of the rectangular waveguide

To obtain a qualitative understanding of these modes, our analysis is based on Marcatili's method for 3D optical waveguides. We consider the 3D waveguide as two three-layer planar waveguides in x and y directions. So, there are no pure TE and TM modes, but the resembling TEM modes. On one hand, if the electric field is polarized in the y direction and the principal field components are E_y and H_x, this mode is called E^y_{mn} mode. On the other hand, if the electric field is polarized in the x direction and principal field components are E_x and H_y, this mode is called E^x_{mn} mode. Here, m and n are integers corresponding to the number of peaks of the optical power in x and y directions, respectively.

Since the extinction coefficient is rather smaller than the real refractive index, we can use the perturbation method to analyze the absorption property of LHM rectangular waveguide.

Without loss of generality here we assume that the influence of extinction coefficient on effective refractive index is negligible. It is also assumed that the electric and magnetic fields are confined to the core, which decay exponentially in the cladding and are negligible in the shaded regions of Fig. 1. We obtain the following dispersion equations [9] for E^y_{mn} mode:

$$\gamma_{1y}b = n\pi + \arctan\left[\left(\frac{\varepsilon_1}{\varepsilon_2}\right)\frac{\gamma_{2y}}{\gamma_{1y}}\right] + \arctan\left[\left(\frac{\varepsilon_1}{\varepsilon_2}\right)\frac{\gamma_{4y}}{\gamma_{1y}}\right] \tag{1}$$

$$\gamma_{1x}a = m\pi + \arctan\left[\left(\frac{\mu_{N_1}}{\mu_3}\right)\frac{\gamma_{3x}}{\gamma_{1x}}\right] + \arctan\left[\left(\frac{\mu_{N_1}}{\mu_5}\right)\frac{\gamma_{5x}}{\gamma_{1x}}\right] \tag{2}$$

and for E^x_{mn} mode

$$\gamma_{1y}b = n\pi + \arctan\left[\left(\frac{\mu_1}{\mu_2}\right)\frac{\gamma_{2y}}{\gamma_{1y}}\right] + \arctan\left[\left(\frac{\mu_1}{\mu_4}\right)\frac{\gamma_{4y}}{\gamma_{1y}}\right] \tag{3}$$

$$\gamma_{1x}a = m\pi + \arctan\left[\left(\frac{\varepsilon_{N_1}}{\varepsilon_3}\right)\frac{\gamma_{3x}}{\gamma_{1x}}\right] + \arctan\left[\left(\frac{\varepsilon_{N_1}}{\varepsilon_5}\right)\frac{\gamma_{5x}}{\gamma_{1x}}\right] \tag{4}$$

where $\gamma_{1y}=(k_0^2n_1^2-\beta_1^2)^{1/2}$, $\gamma_{2y}=(\beta_1^2-k_0^2n_2^2)^{1/2}$, $\gamma_{4y}=(\beta_1^2-k_0^2n_4^2)^{1/2}$, $\gamma_{1x}=(k_0^2N_1^2-\beta^2)^{1/2}$, $\gamma_{3x}=(\beta^2-k_0^2n_3^2)^{1/2}$, and $\gamma_{5x}=(\beta^2-k_0^2n_5^2)^{1/2}$. β_1 is the propagating constant in three-layer planar waveguide in y direction and $\beta_1=k_0N_1$ where N_1 denotes its effective refractive index. β is the propagating constant in three-layer planar waveguide in x direction and $\beta=k_0N$, where N denotes its effective refractive index.

Then we take the extinction coefficient ($\kappa \neq 0$) into account and make use of the results of absorption of planar waveguides [9] to derive some expressions of the attenuation coefficient.

For E^y_{mn} mode, equation (2) is similar to the dispersion equation of TE modes in three-layer planar waveguide in x direction. So, the attenuation coefficient expression can be obtained as follows:

$$\alpha(E^y_{mn}) = \alpha_{TE} = \frac{1}{Na_{eff}}\left[N_1\alpha_{TM}\left(a+\frac{\mu_{N_1}\mu_3\gamma_{3x}}{\mu_3^2\gamma_{1x}^2+\mu_{N_1}^2\gamma_{3x}^2}+\frac{\mu_{N_1}\mu_3\gamma_{5x}}{\mu_3^2\gamma_{1x}^2+\mu_{N_1}^2\gamma_{5x}^2}\right)+n_3\alpha_3\left(\frac{\mu_{N_1}\mu_3\gamma_{1x}^2}{\gamma_2(\mu_3^2\gamma_{1x}^2+\mu_{N_1}^2\gamma_{3x}^2)}\right)+n_5\alpha_5\left(\frac{\mu_{N_1}\mu_5\gamma_1^2}{\gamma_3(\mu_5^2\gamma_{1x}^2+\mu_{N_1}^2\gamma_{5x}^2)}\right)\right] \tag{5}$$

where a_{eff} is the effective core width of the assumed three-layer planar waveguide in x direction, and satisfy the following equation

$$a_{eff} = a + \frac{\mu_{N_1}\mu_3(\gamma_{1x}^2+\gamma_{3x}^2)}{\gamma_{3x}(\mu_3^2\gamma_{1x}^2+\mu_{N_1}^2\gamma_{3x}^2)} + \frac{\mu_{N_1}\mu_5(\gamma_{1x}^2+\gamma_{5x}^2)}{\gamma_{5x}(\mu_5^2\gamma_{1x}^2+\mu_{N_1}^2\gamma_{5x}^2)} \tag{6}$$

In equation (5), α_{TM} is the attenuation coefficient of TM modes in y direction with core width of b, and it can be obtained:

$$\alpha_{TM} = \frac{1}{N_1 b_{eff}}\left[n_1\alpha_1\left(b+\frac{\varepsilon_1\varepsilon_2\gamma_{2y}}{\varepsilon_2^2\gamma_{1y}^2+\varepsilon_1^2\gamma_{2y}^2}+\frac{\varepsilon_1\varepsilon_4\gamma_{4y}}{\varepsilon_4^2\gamma_{1y}^2+\varepsilon_1^2\gamma_{4y}^2}\right)+n_2\alpha_2\left(\frac{\varepsilon_1\varepsilon_2\gamma_{1y}^2}{\gamma_{2y}(\varepsilon_2^2\gamma_{1y}^2+\varepsilon_1^2\gamma_{2y}^2)}\right)+n_4\alpha_4\left(\frac{\varepsilon_1\varepsilon_4\gamma_{1y}^2}{\gamma_{4y}(\varepsilon_4^2\gamma_{1y}^2+\varepsilon_1^2\gamma_{4y}^2)}\right)\right] \tag{7}$$

where b_{eff} is the effective core width of the assumed three-layer planar waveguide in y direction, and satisfy the following equation

$$b_{eff} = b + \frac{\varepsilon_1\varepsilon_2(\gamma_{1y}^2+\gamma_{2y}^2)}{\gamma_{2y}(\varepsilon_2^2\gamma_{1y}^2+\varepsilon_1^2\gamma_{2y}^2)} + \frac{\varepsilon_1\varepsilon_4(\gamma_{1y}^2+\gamma_{4y}^2)}{\gamma_{4y}(\varepsilon_4^2\gamma_{1y}^2+\varepsilon_1^2\gamma_{4y}^2)} \tag{8}$$

For E^x_{mn} mode, the dispersion equation of TM modes in three-layer planar waveguide in x direction is similar to equation (5). So, the attenuation coefficient expression can be obtained as follows:

$$\alpha(E^x_{mn}) = \alpha_{TM} = \frac{1}{Na_{eff}}\left[N_1\alpha_{TE}\left(a+\frac{\varepsilon_{N_1}\varepsilon_3\gamma_{3x}}{\varepsilon_3^2\gamma_{1x}^2+\mu_{N_1}^2\gamma_{3x}^2}+\frac{\varepsilon_{N_1}\varepsilon_3\gamma_{5x}}{\varepsilon_3^2\gamma_{1x}^2+\varepsilon_{N_1}^2\gamma_{5x}^2}\right)+n_3\alpha_3\left(\frac{\varepsilon_{N_1}\varepsilon_3\gamma_{1x}^2}{\gamma_2(\varepsilon_3^2\gamma_{1x}^2+\varepsilon_{N_1}^2\gamma_{3x}^2)}\right)+n_5\alpha_5\left(\frac{\varepsilon_{N_1}\varepsilon_5\gamma_1^2}{\gamma_3(\varepsilon_5^2\gamma_{1x}^2+\varepsilon_{N_1}^2\gamma_{5x}^2)}\right)\right] \tag{9}$$

where a_{eff} is the effective core width of the assumed three-layer planar waveguide in x direction, and satisfy the following equation

$$a_{eff} = a + \frac{\varepsilon_{N_1}\varepsilon_3(\gamma_{1x}^2+\gamma_{3x}^2)}{\gamma_{3x}(\varepsilon_3^2\gamma_{1x}^2+\varepsilon_{N_1}^2\gamma_{3x}^2)} + \frac{\varepsilon_{N_1}\varepsilon_5(\gamma_{1x}^2+\gamma_{5x}^2)}{\gamma_{5x}(\varepsilon_5^2\gamma_{1x}^2+\varepsilon_{N_1}^2\gamma_{5x}^2)} \tag{10}$$

Similar to equation (9), α_{TE} is the attenuation coefficient of TE modes in y direction with core width of b, and we can obtain

$$\alpha_{TE} = \frac{1}{N_1 b_{eff}}\left[n_1\alpha_1\left(b+\frac{\mu_1\mu_2\gamma_{2y}}{\mu_2^2\gamma_{1y}^2+\mu_1^2\gamma_{2y}^2}+\frac{\mu_1\mu_4\gamma_{4y}}{\mu_4^2\gamma_{1y}^2+\mu_1^2\gamma_{4y}^2}\right)+n_2\alpha_2\left(\frac{\mu_1\mu_2\gamma_{1y}^2}{\gamma_{2y}(\mu_2^2\gamma_{1y}^2+\mu_1^2\gamma_{2y}^2)}\right)+n_4\alpha_4\left(\frac{\mu_1\mu_4\gamma_{1y}^2}{\gamma_{4y}(\mu_4^2\gamma_{1y}^2+\mu_1^2\gamma_{4y}^2)}\right)\right] \tag{11}$$

where b_{eff} is the effective core width of the assumed three-layer planar waveguide in y direction, and satisfy the following equation

$$b_{eff} = b + \frac{\mu_1\mu_2(\gamma_{1y}^2+\gamma_{2y}^2)}{\gamma_{2y}(\mu_2^2\gamma_{1y}^2+\mu_1^2\gamma_{3y}^2)} + \frac{\mu_1\mu_4(\gamma_{1y}^2+\gamma_{4y}^2)}{\gamma_{4y}(\mu_4^2\gamma_{1y}^2+\mu_1^2\gamma_{4y}^2)} \tag{12}$$

3 Numerical Results

The propagation modes are more strictly confined in rectangular waveguides than in planar waveguides. Due to the restriction of the dispersion equations, the LHM waveguide can not support the E^y_{0n}, E^y_{m0}, E^x_{0n} or E^x_{m0} modes and its absorption property is quite different from that of RHM (right-handed material) rectangular waveguide in the following two aspects.

3.1 Different a/b and Different Attenuation Curves

For absorption of LHM rectangular waveguide, we can derive the numerical results of the attenuation coefficients calculated by using the expressions (5) and (9). The guidance conditions of modes are derived, and the transcendental equation for guided E^y_{mn} wave is solved numerically. We choose the wavelength $\lambda_0 = 1.5\ \mu m$, $n_1 = -1.594$, $n_2 = n_3 = n_4 = n_5 = 1.583$, and $\mu_1/\mu_2 = \mu_1/\mu_3 = \mu_1/\mu_4 = \mu_1/\mu_5 = -1$. When calculating the attenuation coefficient α, it is necessary to take the condition of guided modes' existence into account ($1.583^2 < N^2 < 1.594^2$) and then discuss the results. We also choose $\alpha_1 = 5\times10^{-5}\ \mu m^{-1}$, $\alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 5\times10^{-6}\ \mu m^{-1}$, when a/b is varying, α is changing dramatically. Three cases are investigated ($a = 1.5b$, $a = 1.75b$ and $a = 2b$). Take the E^y_{11} mode as an example, the attenuation coefficient curves are shown in Fig. 2. The cases as above three ones in RHM rectangular waveguide are also considered in Fig. 3 and all the parameters are chosen as above except that $n_1 = 1.594$.

It can be found that the attenuation curves of LHM and RHM waveguide have different orders and different shapes. In LHM waveguide, the attenuation coefficient changes from large to small as a/b value grows. But for RHM waveguide, the attenuation coefficient changes from small to large as a/b value grows. However, it is also noticed that the

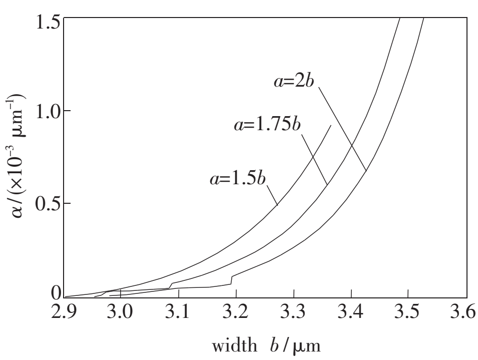

Fig. 2 Attenuation curves of different a/b in LHM waveguide for E^y_{11} mode

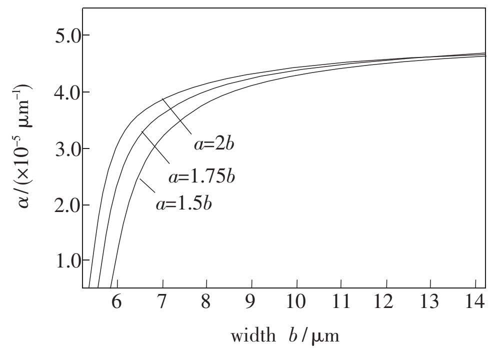

Fig. 3 Attenuation curves of different a/b in RHM waveguide for E^y_{11} mode

general attenuation coefficients of LHM waveguide are larger than RHM waveguide, which shows stronger absorption of the light than that of the latter.

3.2 Different Propagating Modes and Different Attenuation Curves

We choose $\alpha_1 = 5\times10^{-5}\ \mu m^{-1}$, $\alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 5\times10^{-6}\ \mu m^{-1}$, the attenuation curves of E^y_{12}, E^y_{13}, E^y_{14} modes in LHM waveguide when $a = 2b$ are as shown in Fig. 4. When calculating the attenuation coefficient, it is necessary to take the condition of guided modes' existence into account ($1.583^2 < N^2 < 1.594^2$) and then discuss the results. We find that, in LHM waveguide there are an absorption peak for every mode and an attenuation trough in a small neighborhood of this peak. They are so close to each other that slight width changes will lead to distinct attenuation coefficient changes. In addition to this, higher-order modes possess larger attenuation coefficients than that of lower-order modes. Because the absorption property of LHM waveguide is sensitive to the width, it is necessary to choose proper

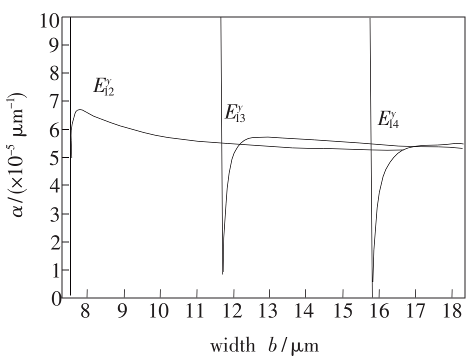

Fig. 4 Attenuation curves of different modes in LHM waveguide

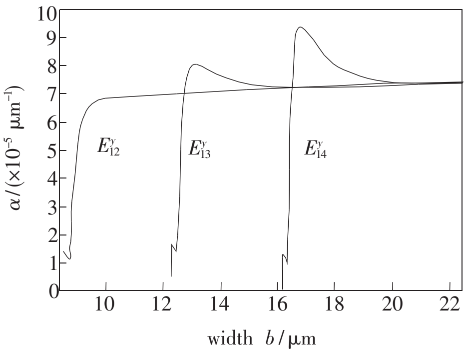

Fig. 5 Attenuation curves of different modes in RHM waveguide

length and width accurately to get different absorption property. This property is rather useful when designing filters or attenuators. But in RHM (n = 1.594) waveguide, the attenuation curves are changing vibrantly and approach the core attenuation coefficient with the increasing of the width b, as shown in Fig. 5. Otherwise, higher-order modes possess smaller attenuation coefficients than that of lower-order modes in RHM waveguide.

4 Conclusion

We investigated the absorption property of the LHM rectangular waveguide by using perturbation method. The results demonstrated a number of unusual properties that differ considerably from those of a RHM waveguide. It is shown that in LHM waveguide, mode's attenuation coefficient changes from large to small as a/b grows. In addition to this, when E_{12}^{y}, E_{13}^{y}, E_{14}^{y} modes are propagating in the same LHM waveguide, higher-order modes possess larger attenuation coefficients than that of lower-order modes. In this case, for every mode there is an absorption peak, and in a small neighborhood of this peak, an attenuation trough can be found.

These properties are of particular relevance to devices such as filters and WDM devices. So, we come into the conclusion that LHM waveguides possess lots of unique and novel properties over RHM waveguide, and it will bring us opportunities to design new devices and have special application.

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左手介质矩形波导的吸收特性

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摘要：采用微扰法研究了左手介质矩形波导的吸收特性。由于色散方程的限制，左手介质波导不能传输基模，它的吸收特性也与右手介质矩形波导明显不同。结果发现，模式的衰减系数随长宽比（a/b）的增加而减小，而在右手介质波导中结果恰恰相反。除此之外，当不同模式的光波在同一左手介质矩形波导中传输时，每一模式都对应着相邻很近的吸收峰和吸收谷。同时高阶模的衰减系数较低阶模的大。但是，当不同模式的光波在同一右手介质矩形波导中传输时，它们的衰减曲线随着波导宽度的增加而振荡的变化，并且趋近于芯层衰减系数。另外一方面，在右手波导中，高阶模式的衰减系数较低阶模式小。

关 键 词：吸收峰；衰减谷；衰减系数；微扰法；矩形波导
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