Article ID: 1000-7032(2007)03-0302-05

The Influence of Driving-field Phase Diffusion on Lasing without Inversion in a Cascade System

ZHU Meng-zheng, ZHAO Chun-ran*,
ZHANG Feng, YIN Xin-guo, DAI Jian-ming
(Department of Physics, Huaibei Coal Industry Teachers College, Huaibei 235000, China)

Abstract The influence of driving field phase fluctuation on lasing without population inversion in a three-level cascade system was studied. In the rotating wave and slowly varying envelope approximations, the density-matrix motion equations for the three-level cascade system, considered the influence of driving-field phase diffusion is solved, and the exact linear analytical solutions of the three-level cascade system with the driving field having the phase fluctuation in the steady state are obtained. The dependence of LWI gain, dispersion and the populations on the probe field detuning and the strengths of coherent pumping respectively, are examined and simulated numerically. It was shown from the results of numerical calculation about the steady-state solutions of three-level cascade system that the gain of lasing without population inversion will be decreased due to driving-field phase-fluctuation. The effect can be compensated by increased driving-field intensities. However, the effect that LWI gain is decreased due to driving-field phase-fluctuation can not be always compensated by increased driving field Rabi frequency. Lasing without population inversion is still obtained even if the linewidth due to driving-field phase-fluctuation is large enough. The presence of the linewidth prevents the cascade system from obtaining a high refractive index along with zero absorption. The cascade system can still exhibit a larger refractive index and zero absorption at the lesser linewidth. The linewidth tends to destroy lasing without population inversion and refractive index enhancement. There is no population inversion for the lasing transition and for the driving transition under the given condition. And the condition without population inversion has nothing to do with the variety of linewidth in the steady-state analytical solutions. That is to say, there is not possibility for the cascade system that a change from lasing with population inversion to lasing without population inversion can occur with the linewidth increasing or with the Rabi frequency of driving-field increasing. The conclusion is very different from that obtained in other inversionless lasing system.

Key words: atomic coherence; lasing without inversion; phase diffusion; cascade system
CLC number: O431.2; O432.12 **PACC:** 4250; 4255 **Document code:** A

1 Introduction

Quantum coherence and interference[1~7] have led to a number of important optical consequences such as lasing without inversion (LWI), electromagnetically induced transparency and subrecoil cooling of atoms. In particular, LWI has attracted much more attention[2~7,9,11~14] due to its important science sense and potentially wide application. However, the phase of the driving field is usually assumed to be fixed in many studies on LWI. In practice, the phase is fluctuant. The phase diffusion leads to loss contributions and to a decay of the coherence[4]. Based on the effects of phase fluctuation

Received date: 2006-10-29; **Revised date:** 2006-12-27
Foundation item: Project supported by the Innovation Project of Science and Technology of Anhui Province (2006KJ003TD); the Scientific Research Sustentation Fund for Young Teachers in the Higher Education Institutions of Anhui Province (2005jq1133)
Biography: ZHU Meng-zheng, male, was born in 1978, Anhui Province, Instructor. His work focuses on the field is nonlinear optics and quantum optics.
*: Corresponding Author; E-mail: zhu97119@yahoo.com.cn, Tel: 13966119640

in an open four-level system with the two driving fields, Zhu *et al* [5] found from the research of an open four-level system with the single driving field that a change from conventional laser to LWI can occur with the linewidth due to phase fluctuation increasing. Zhu *et al* [6] found that variation of the linewidth cannot change the property of the inversionless lasing of the system. In the paper, it is studied that the phase diffusion of driving field is how to influence on lasing without inversion in a three-level cascade system.

2 Model and Linear Solutions

Consider a closed three-level cascade system[7] with the ground state $|3\rangle$ and excited states $|2\rangle$ and $|1\rangle$ as illustrated in Fig. 1. The transition $|2\rangle \to |1\rangle$ of frequency ω_{21} is driven by a laser of frequency ω_d with Rabi frequency $2\Omega_1$. A weak probe laser of frequency ω_p with Rabi frequency $2\Omega_2$ is applied to the transition $|3\rangle \to |2\rangle$. An incoherence pump field of rate 2Λ is applied between ground state $|3\rangle$ and excited state $|2\rangle$. $2\gamma_j$ is the spontaneous decay rate of state $|j\rangle$ $(j = 1$ or $2)$. The transition $|1\rangle \leftrightarrow |3\rangle$ is forbidden. If the probe laser is amplified through the system, lasing can be established on the transition $|2\rangle \to |3\rangle$. For instance, in the H_2 molecule the three-level cascade system is formed by the $X^1\Sigma_g^+$ $(v = 0, j = 0)$ ground state (state $|3\rangle$), the $B^1\Sigma_u^+$ $(v = 0, j = 1)$ excited state (state $|2\rangle$), and the $E, F^1\Sigma_g^+$ $(v = 0, j = 0)$ excited state (state $|1\rangle$). The density-matrix motion equations for the system are given in Ref [7]. Here we redefine $\Gamma_{12} = \gamma_1 + \gamma_2$, $\Gamma_{23} = \gamma_2 + \Lambda$, and $\Gamma_{13} = \gamma_1 + \Lambda$. For convenience of calculation, we have assumed that Ω_2 in Eq (1) is real.

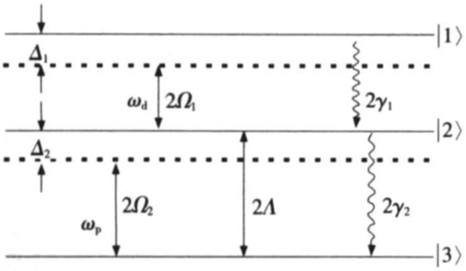

Fig 1 A three-level cascade system.

Let $\varphi(t)$ represents the phase fluctuation of the driving field, i.e.,

$$\Omega_1 = \Omega_{10}\exp[i\varphi(t)] \tag{1}$$

The phase is characterized by the following random equation of motion[8]:

$$\varphi'(t) = u(t) \tag{2}$$

with zero average, i.e., $\langle u(t)\rangle = 0$. Here $u(t)$ is a δ-correlated Langevin-noise term, whose diffusion coefficient gives the linewidth $2R_{ph}$ of the driving field, i.e.,

$$\langle u(t)u(t')\rangle = 2R_{ph}\,\delta(t - t') \tag{3}$$

Eq (1) is therefore a stochastic equations with multiplicative white noise, which gives rise to noise-induced drift terms that alter the semiclassical evolution of the system. In order to clarify the influence of the finite linewidth, we redefine the variables and as follows

$$\rho_{12} = \rho'_{12}\exp[i\varphi(t)], \quad \rho_{13} = \rho'_{13}\exp[i\varphi(t)] \tag{4}$$

Consequently, the density matrix motion Eq (3) in Ref [7] should be averaged over the randomly fluctuating phase. That is to say, the density matrix element ρ_{ij} $(i, j = 1, 2, 3)$ must be replaced by $\langle\rho_{ij}\rangle$. We derive the semiclassical set of equations for the stochastic averaged values of $\langle\rho_{jj}\rangle$ $(j = 1 \sim 3)$ and $\langle\rho_{12}\rangle$ correct to the zeroth order of the probe field, and for the averaged values of the polarizations $\langle\rho_{13}\rangle$ and $\langle\rho_{23}\rangle$ correct to the first order of the probe field by using the method in Refs [5, 9], i.e.:

$$\langle\dot\rho_{12}\rangle = -(\Gamma'_{12} + i\Delta_1)\langle\rho_{12}\rangle + i\Omega_{10}(\langle\rho_{22}\rangle - \langle\rho_{11}\rangle) \tag{5a}$$

$$\langle\dot\rho_{13}\rangle = -[\Gamma'_{13} + i(\Delta_1 + \Delta_2)]\langle\rho_{13}\rangle + i\Omega_{10}\langle\rho_{23}\rangle - i\Omega_2\langle\rho_{12}\rangle \tag{5b}$$

where $\Gamma'_{12} = \Gamma_{12} + R_{ph}$ and $\Gamma'_{13} = \Gamma_{13} + R_{ph}$. Comparing Eq (3) in Ref [7] and the Eqs (5), one can find that the phase fluctuation leads to loss contributions.

For the steady state, $\langle\rho_{11}\rangle + \langle\rho_{22}\rangle + \langle\rho_{33}\rangle \equiv 1$. We obtain the steady-state linear solutions for population differences $P_{21} \equiv \langle\rho_{22}\rangle - \langle\rho_{11}\rangle$, $P_{32} \equiv \langle\rho_{33}\rangle - \langle\rho_{22}\rangle$, and the polarization $\langle\rho_{23}\rangle$:

$$P_{21} = \eta_6\eta_9 \tag{6a}$$

$$P_{32} = \eta_8\eta_9 \tag{6b}$$

$$\mathrm{Im}\langle\rho_{23}\rangle = \Omega_2[(\eta_2\eta_4 - \eta_1\eta_3)\eta_5 P_{21} + \eta_1 P_{32}]\eta_{10} \tag{6c}$$

$$\mathrm{Re}\langle\rho_{23}\rangle = \Omega_2[(\eta_1\eta_4 + \eta_2\eta_3)\eta_5 P_{21} - \eta_2 P_{32}]\eta_{10} \tag{6d}$$

Detailed expressions for $\eta_i\,(i = 1\sim 10)$ in the Eqs (6) are as follows

$$\eta_1 = \eta_0\Gamma'_{13} + \Gamma_{23},\quad \eta_2 = \eta_0 D - \Delta_2,$$

$$\eta_3 = D\Delta_1 - \Gamma'_{13}\Gamma'_{12},\quad \eta_4 = D\Gamma'_{12} + \Delta_1\Gamma'_{13}$$

$$\eta_5 = \eta_0/\eta,\quad \eta_6 = \Lambda/(\gamma_2 - \Lambda),$$

$$\eta_7 = 3\Gamma'_{12}\Omega_{10}^2/(\eta\gamma_1) + 2,\quad \eta_8 = (\eta_7 + 1)/3,$$

$$\eta_9 = (\eta_6\eta_7 + \eta_8)^{-1},\quad \eta_{10} = (\eta_1^2 + \eta_2^2)^{-1}$$

Here $D = \Delta_1 + \Delta_2$, $\eta_0 = \Omega_{10}^2/[D^2 + \Gamma'^2_{13}]$, and $\eta = \Delta_1^2 + \Gamma'^2_{12}$.

3 Discussion and Results

The gain coefficient of the probe field is proportional to $\mathrm{Im}\langle\rho_{32}\rangle$. If $\mathrm{Im}\langle\rho_{32}\rangle > 0$, the system exhibits gain for the probe field; if $\mathrm{Im}\langle\rho_{32}\rangle < 0$, the probe field is attenuated. Furthermore, the dispersion is determined by $\mathrm{Re}\langle\rho_{32}\rangle$, $\mathrm{Re}\langle\rho_{32}\rangle > 0$ corresponds to the red shift of the frequency of the probe field, $\mathrm{Re}\langle\rho_{32}\rangle < 0$ shows the blue shift[10]. The refractive index of medium is proportional to $\mathrm{Re}\langle\rho_{32}\rangle$. $\langle\rho_{32}\rangle = \langle\rho_{23}\rangle^*$. If the inequation $\gamma_2 < \Lambda$ is satisfied in the three-level cascade system, one can find from the expressions of $\eta_n\,(n = 6\sim 9)$ that $\eta_n > 0$. Therefore, the inequations $\langle\rho_{11}\rangle < \langle\rho_{22}\rangle < \langle\rho_{33}\rangle$ can be always satisfied under the condition that $\gamma_2 < \Lambda$. That is to say, there is no possibility for the cascade system that a change from lasing with population inversion to lasing without population inversion can occur with the linewidth increasing or with the Rabi frequency of driving-field increasing. The conclusion is very different from that obtained in other inversionless lasing system[5, 11, 12].

The plots of $\mathrm{Im}\langle\rho_{32}\rangle/\Omega_2$ and $\mathrm{Re}\langle\rho_{32}\rangle/\Omega_2$ versus the probe field detuning Δ_2 are presented, as shown in Fig. 2 by using the numerical calculation result from Eq. (6). Values of parameters are $\Lambda = 0.9995$, $\gamma_2 = 1$, $\gamma_1 = 0.28$, $\Omega_{10} = 10$, $\Delta_1 = 0$. The populations in the three levels is not dependent on Δ_2 because η_n s$(n = 6\sim 9)$ are not as the functions of probe-field detuning. Therefore, the curves of the populations in three levels vs probe-field detuning are not given. It is found from Fig. 2 that

1. LWI can be obtained, and the gain in lasing without inversion will decrease as the linewidth increases. 2. The cascade system can still exhibit a larger refractive index and zero absorption at the lesser linewidth. Therefore, the linewidth tends to destroy LWI and refractive index enhancement. The conclusions accord with those in other system[5, 13, 14].

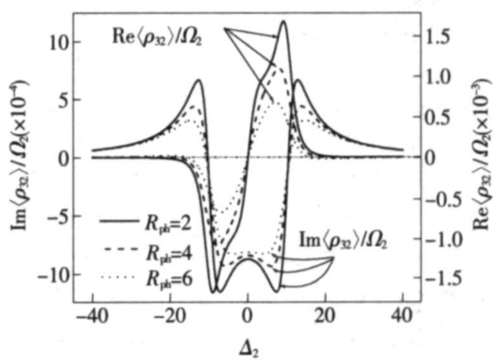

Fig. 2 $\mathrm{Im}\langle\rho_{32}\rangle/\Omega_2$ and $\mathrm{Re}\langle\rho_{32}\rangle/\Omega_2$ *vs* Δ_2.

Fig. 3 illustrates the curves of gain, population and population difference versus Ω_{10} for various R_{ph}, with the same values of system parameters in Fig. 2 but $\Delta_2 = -12.74$. The ladder system exhibits gain for the probe laser even if Ω_{10} is sufficiently little. At the moment the incoherence pump field between ground state $|3\rangle$ and excited state $|2\rangle$ plays a crucial role in obtaining LWI. The maximum LWI gain occurs at a moderate Ω_{10}. Gain decreases till absorption occurs if Ω_{10} increases unceasingly. The characteristic is different from that in other three- or four-level system. The LWI gain for a monochromatic driving field ($R_{\mathrm{ph}} = 0$) is larger than that considered the driving-field phase fluctuation. LWI gain will be decreased due to driving-field phase fluctuation. However, lasing without population is still obtained even if the linewidth is large enough. The effect can be compensated by increased driving-field intensities when Ω_{10} is not sufficiently large[13]. However, one found from Fig. 3 that the effect that LWI gain is decreased due to driving-field phase fluctuation can not be always compensated by increased Ω_{10} value. It is not difficult to comprehend that the population of level

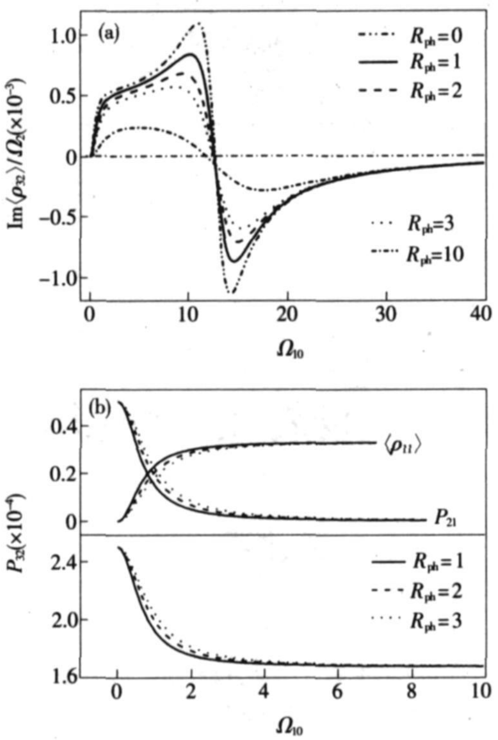

Fig 3 Im $\langle\rho_{32}\rangle/\Omega_2$, Re $\langle\rho_{32}\rangle/\Omega_2$ and the populations *vs* Ω_{10}.

$|1\rangle$ increases as Rabi frequency of the driving field increases. With the linewith increasing, the population differences (P_{32} and P_{21}) increase obviously, nevertheless the population of level $|1\rangle$ decreases evidently when the driving field intensity is with proper magnitude.

4 Conclusion

We have given an exact steady linear analytical solution of the three-level cascade system with the driving field having the phase fluctuation. We find from the numerical results of Eq. (6) that 1. the phase diffusion leads to loss contributions and a decay of the coherent trapping state. LWI gain will be decreased because of the presence of driving-field phase-fluctuation. The effect can be compensated by increased driving-field intensities. However, the effect that LWI gain is decreased due to driving-field phase fluctuation can not be always compensated by increased Ω_{10} value. 2. The presence of the linewidth prevents the cascade system from obtaining a high refractive index along with zero absorption. Therefore, the linewidth tends to destroy both lasing without inversion and refractive index enhancement. 3. The condition without population inversion has nothing to do with the variety of linewidth in steady-state analytical solution. Accordingly, the variation of finite linewidth cannot change the property of the inversionless lasing of the system.

References

[1] Fleischhauer M, Imamoglu A, Marangos J P. Electromagnetically induced transparency: Optics in coherent media [J]. *Rev. Mod. Phys.*, 2005, **77**(2): 633-672.

[2] Gonzalo I, Ant n M A, Carre o F. Squeezing in a Λ-type three-level atom via spontaneously generated coherence [J]. *Phys. Rev.* A, 2005, **72**(3): 033809.

[3] Mompart J, Corbalan R. Lasing without inversion [J]. *J. Opt.* B: *Quant. Semiclas. Opt.*, 2000, **2**(3): R4-27.

[4] Sultana S, Zubairy M S. Effect of finite bandwidth on refractive-index enhancement and lasing without inversion [J]. *Phys. Rev.* A, 1994, **49**(1): 438-448.

[5] Zhu M Z, Zhao C R, Fan X J, *et al.* Lasing without or with inversion in an open four-level system with a phase-fluctuation driving field [J]. *J. Mod. Opt.*, 2006, **53**(15): 2101-2110.

[6] Zhu Mengzheng, Zhao Chunran, Yin Xinguo. Effects of driving-field phase fluctuation on an open four-level system [J]. *Acta Photonica Sinica (光子学报)*, 2006, **35**(10): 1602-1607 (in Chinese).

[7] Das R K, Sanyal S, Dastidar K R. Control of amplification without population inversion in H_2 and Li_2 molecules: three level ladder, V and Λ systems [J]. *Eur. Phys. J. D*, 2005, **32**(1): 95-112.

[8] Louisell W H. *Quantum Statistical Properties of Radiation* [M]. New York: Wiley, 1973.

[9] Fleischhauer M, Scully M O. Quantum sensitivity limits of an optical magnetometer based on atomic phase coherence [J]. *Phys. Rev.* A, 1994, **49**(3): 1973-1986.

[10] Sargent M, Scully M O, Lamb W E. *Laser Physics* [M]. New York: Addison-Wesley, 1974.

[11] Gong S Q, Xu Z Z, Zhang Z Q. Change from an inversion laser to a noninversion laser due to a phase-fluctuation effect

[J]. *Phys. Rev.* A, 1995, **52**(6): 4787-4790.

[12] Gong S Q, Du S D, Xu Z Z. Nonlinear theory of lasing with or without inversion in a simple three-level atomic system [J]. *Opt. Commun.*, 1996, **130**(4-6): 249-254.

[13] Fleischhauer M, Lukin M D, Nikonov D E, *et al.* Influence of pump-field phase diffusion on laser gain in a double-non-inversion laser [J]. *Opt. Commun.*, 1994, **110**(3-4): 351-357.

[14] Scully M O, Zhu S Y. Ultra-large index of refraction via quantum interference [J]. *Opt. Commun.*, 1992, **87**(3): 134-138.

驱动场相位扩散对三能级梯型系统无反转激光的影响

朱孟正, 赵春然*, 张 峰, 尹新国, 戴建明
(淮北煤炭师范学院 物理系, 安徽 淮北 235000)

摘要: 在旋波、慢变振幅近似下,求解考虑了驱动场相位扩散后的系统密度矩阵运动方程,并给出了这个三能级梯型系统稳态线性解析解。利用得到的稳态线性解析解分析驱动场相位扩散是如何影响该系统输出无反转激光的。对稳态线性解析解数值计算的结果显示:由于驱动场相位扩散会导致无反转激光增益减小;即使由于驱动场相位扩散引起的线宽足够大,在该系统中仍能够获得无反转激光;线宽往往是破坏无反转激光产生和折射率的提高;因驱动场相位扩散导致无反转激光增益的减小,并不是总能够通过增大驱动场的 Rabi 频率得到补偿。

关 键 词: 原子相干; 无粒子数反转激光; 相位扩散; 梯型系统
中图分类号: O431.2; O432.12 **PACC:** 4250; 4255 文献标识码: A
文 章 编 号: 1000-7032(2007)03-0302-05

收稿日期: 2006-10-29; 修订日期: 2006-12-27
基金项目: 安徽省科技创新团队项目(2006KJ003TD); 安徽省高等学校青年教师科研基金(2005jq1133)资助项目
作者简介: 朱孟正(1978-), 男, 安徽无为人, 讲师, 主要从事非线性光学和量子光学领域的研究。
*: 通讯联系人; E-mail: zhu97119@yahoo.com.cn, Tel: 13966119640